IS 9300-2 (1989): Statistical models for industrial applications, Part 2: Continuous models [MSD 3: Statistical Methods for Quality and Reliability]

BLANK PAGE

AMENDMENT NO. 1 SEPTEMBER 1993
TO
IS 9300 (Part 2) : 1989 STATISTICAL MODELS FOR INDUSTRIAL APPLICATIONS

PART 2 CONTINUOUS MODELS

(First Revision)

(*Page* 9, *clause* **9.4**) — Insert the following new clause after **9.4**:

'**9.4.1** *Example*

The following data gives the running time (in hours) of the 40 head boxes. Test whether the data follows the Weibull distribution:

249	259	844	65
303	309	121	32
649	21	146	43
23	104	99	301
130	52	584	21
411	177	583	508
643	281	173	9
180	169	248	42
128	173	524	883
212	547	31	85

The above data is arranged in the form of a frequency table as shown in Table 10.

The next step is to calculate the expected frequencies for each class interval under assumption that the data follows Weibull distribution. For this purpose, one has to estimate the parameters β and λ of the Weibull model. For estimating the parameters, the natural logarithmic values of each of the x_i's are obtained and thereafter the mean and the standard deviation of these x_i values are calculated.

For this example, mean ($\bar{y}$) = 5·042 1; and

standard deviation (s_y) = 1·16

where, $y_i = \log x_i$.

The estimate of the parameters are then obtained from the following expression:

$$\beta = \frac{\pi}{s_y \sqrt{6}} \text{ and } \lambda = \exp \left[-\left(\bar{y} + \frac{0{\cdot}572\ 26}{\beta} \right) \right]$$

For this example, $\beta = 1{\cdot}106\ 1$ and $\lambda = 0{\cdot}003\ 85$

Using the above estimates of the parameters in the frequency distribution of the Weibull model, namely,

$F(x) = 1 - \exp\left[-(\lambda x)^{\beta}\right]$, where, $\beta = 1{\cdot}106\ 1$ and $\lambda = 0{\cdot}003\ 85$, the expected frequency for each class interval may be obtained as given in Table 10.

Goodness of Fit

The calculated value of χ^2 for the example is 1·369 (*see* Table 11). The tabulated value of χ^2 for 2 degrees of freedom at 5 percent level of significance is 5·99. Since the calculated value is less than the tabulated value, the null hypothesis that the data follows Weibull distribution is accepted.

Table 10 Frequency Table for Running Time of Head Boxes

(*Clause* 9.4.1)

Class Intervals	Frequency (O_i)	Upper Limit (x)	$(\lambda x)^{\beta}$	$F(x)$	$F(x_1) - F(x_2)$	Expected Frequency (E_i)
0-100	12	100	0·347 9	0·293 8	0·293 8	11·75
101-200	10	200	0·748 9	0·527 1	0·223 3	9·33
201-300	5	300	1·173 0	0·690 6	0·163 5	6·54
301-400	3	400	1·612 0	0·800 5	0·109 9	4·40
401-500	3	500	2·064 0	0·873 1	0·072 6	2·90
501-600	2	600	2·525 0	0·919 9	0·046 8	1·87
601-700	2	700	2·994 0	0·949 9	0·030 0	1·20
701-800	2	800	3·470 0	0·968 9	0·019 0	0·76
801-900	1	900	3·953 0	0·980 8	0·011 9	0·48
901 & above	0	—	—	1·000 0	0·019 2	0·77

Table 11 Observed and Expected Frequencies

(*Clause* 9.4.1)

Observed Frequencies (O_i)		Expected Frequencies (E_i)		($O_i - E_i$)	$\frac{(O_i - E_i)^2}{E_i}$
12		11·75		0·25	0
10		9·33		0·67	0·048
5		6·54		− 1·54	0·363
3	6	4·40	7·30	− 1·30	0·232
3		2·90			
2	7	1·87	5 08	− 1·92	0·726
2		1·20			
2		0·76			
1		0·48			
0		0·77			
				Total	1·369

(MSD 3)

Printed at Simco Printing Press, Delhi

AMENDMENT NO. 2 SEPTEMBER 2000
TO
IS 9300 (PART 2) : 1989 STATISTICAL MODELS FOR INDUSTRIAL APPLICATIONS

PART 2 CONTINUOUS MODELS

(First Revision)

(*Page* 7, *clause* 7.4) — Insert the following clause at the end of 7.4:

7.5 Fitting a Gamma Model

7.5.1 *Example* — In a manufacturing process of jute proudcts, breaker card stage is the first stage of filamentation for the subsequent processing. From a sample of 10 cm carded sliver (strand of carded raw jute is called sliver), single fibres were segregated and their lengths were measured which are grouped in a frequency table (*see* Table 6).

Fit a gamma model to the above data and test its goodness of fit.

Table 6 Length of Fibres (mm) of Carded Silver

Class Interval	Frequency
0-5	350
5-10	575
10-15	500
15-20	325
20-25	215
25-30	135
30-35	50
35-40	25
45-50	5
Total	2 190

The mean $\bar{x}$ and variance s^2 as calculated from the frequency table are:

$\bar{x}$ = 13.20 mm and s^2 = 72.63 mm

The parameters of the gamma model η and λ can be calculated by solving the following conditions:

$$\bar{x} = \eta/\lambda \text{ and } s^2 = \eta/\lambda^2$$

$$\lambda = \bar{x}/s^2 = 13.20/72.63 = 0.181\,7$$

$$\text{and } \eta = \lambda\bar{x} = 2.399 = 2.40 \text{ (approx)}$$

The next step is to calculate the expected frequencies (e_i) based on the assumption that the above frequency distribution is coming from a gamma distribution. The steps are described in Table 7.

The probability for each class interval is obtained from the table of the Incomplete Gamma Function.

Table 7 Expected Frequencies Based on Gamma Distribution

Class Interval (x_L - x_u)	x	Γ (0.117 3 x, 1.4)	Probability ($x_L < x < x_u$)	Expected Frequency(e_i)
(1)	(2)	(3)	(4)	(5) = 2 190×(4)
0-5	5	0.149	0.149	326.31
5-10	10	0.424	0.275	602.25
10-15	15	0.662	0.238	521.22
15-20	20	0.815	0.153	335.07
20-25	25	0.901	0.086	188.34
25-30	30	0.955	0.054	118.26
30-35	35	0.977	0.022	48.18
35-40	40	0.989	0.012	26.28
40-45	45	0.995	0.006	13.14
45-50	50	0.998	0.003	6.57

Goodness of Fit from χ^2 Test : After calculating expected frequencies (e_i) for each class interval, their closeness with the observed frequencies (o_i) are tested with the help of χ^2 test (*see* Table 8).

Table 8 Calculations for This χ^2 Test

Class Interval	Observed Frequencies (O_i)	Expected Frequency (e_i)	$(O_i-e_i)^2/e_i$
0-5	350	326.31	1.72
5-10	575	602.22	1.23
10-15	500	521.22	0.86
15-20	325	335.07	0.30
20-25	215	188.34	3.77
25-30	135	118.26	2.37
30-35	50	48.18	0.07.
35-40	25	26.28	0.06
40-45	10	13.14	0.75
45-50	5	6.57	0.37
			Total $\chi2$ = 11.50

The total number of classes is 10. Three degrees of freedom are apportioned for the estimation of mean, standard deviation and for total frequency. Thus calculated value of χ^2 is compared with the tabulated value [*see* IS 6200 (Part 2) : 1977] of χ^2 = 14.07 for 7 degrees of freedom at 5 percent level of significance. Since the calculated value is less than the tabulated value the fit can be taken as good one.

(*Page* 8, *Tables* 6, 7, 8 *and* 9) — Table 6, Table 7, Table 8, Table 9 may be replaced by Table 9, Table 10, Table 11, Table 12 respectively.

(*Page* 8, *clause* 8.5) — In line 5, reference to Table 6 may be replaced by Table 9.

(*Page* 8, *clause* 8.5.1) —In line 3, reference to Table 7 may be replaced by Table 10.

(*Page* 8, *clause* 8.5.2) — In line 4, reference to Table 8 may be replaced by Table 11.

(*Page* 8, *clause* 8.5.3) — In line 2, reference to Table 9 may be replaced by Table 12.

(*Page* 1, *clause* 9.4.1, *Amendment No.* 1) — In line 15, reference to Table 10 may be replaced by Table 13.

(*Page* 2, *clause* 9.4.1, *Amendment No.* 1) — Reference to Table 11 may be replaced by Table 14.

(*Page* 2, *Table* 10, *Amendment No.* 1) — Table 10 may be replaced by Table 13.

(*Page* 3, *Table* 11, *Amendment No.* 1) — Table 11 may be replaced by Table 14.

(MSD 3)

Printed at Simco Printing Press, Delhi

Indian Standard

STATISTICAL MODELS FOR INDUSTRIAL APPLICATIONS

PART 2 CONTINUOUS MODELS

(First Revision)

1 SCOPE

1.1 This standard (Part 2) describes the most commonly used continuous statistical models, their potentiality and application in industries with suitable illustrations.

The models covered in this standard are normal, exponential, gamma, Weibull and lognormal.

2 REFERENCES

2.1 The following Indian Standards are necessary adjuncts to this standard:

IS No.	*Title*
IS 7920 : 1985	Statistical vocabulary (*first revision*)
IS 9300 (Part 1) : 1979	Statistical models for industrial applications : Part 1 Discrete models

3 TERMINOLOGY

3.1 For the purpose of this standard, the definitions given in IS 7920 : 1985 shall apply.

4 PROBABILITY DISTRIBUTIONS

4.1 When a random variable X takes continuous values, it is not possible to determine the probability of X taking any particular value. One may only consider the probability of X taking any value within a very small interval of length dx, that is, probability of X lying between x and $(x + dx)$ or between $[x - (dx/2)]$ and $[x + (dx/2)]$ as $\phi(x)\ dx$ where $\phi(x)$ is a continuous function of X and is called the probability density function or simply density funtion of X. The probability density function $\phi(x)$ is always non-negative and corresponds to p_i's in the discrete case:

$$\text{Thus} \int_a^b \phi(x)\ dx = 1$$

where x takes values between the interval (a, b)

4.2 Mean and Variance of Probability Distribution

The mean of the probability distribution is called the expected value of the variable X and denoted by $E(X)$. The variance of the probability distribution is denoted by $V(X)$.

$$E(X) = \int_a^b x\, \phi(x)\ dx$$

and

$$V(X) = \int_a^b [x - E(X)]^2\, \phi(x)\, dx$$

$$\int_a^b x^2\, \phi(x)\, dx - [E(X)]^2$$

5 NORMAL MODEL

5.1 In many practical situations in the industry and in the nature, there is a tendency for the observations to cluster around some central value, and at the same time the frequencies for observations above and below this central value have a declining trend and they taper off as one goes farther and farther.

5.2 A frequency curve obtained in such situations is symmetrical and bell-shaped as shown in Fig. 1. The curve, known as 'Normal Curve' has extensive applications in statistical theory and practice. In practice, many models can be well approximated by the normal model. From the point of view of presentation of data, the important property of the normal model is that a set of data constituting a random sample from such a model can be represented completely by the mean and standard deviation of the sample.

5.3 A normal model has the following properties :

a) It is symmetrical, unimodal and bell-shaped;

b) The values of the mean, median and mode are identical;

c) It is uniquely determined by the two parameters, namely, mean and standard deviation;

d) In the family of normal curves, smaller the standard deviation, higher will be the peak;

e) 95·45 [illegible] will [illegible]

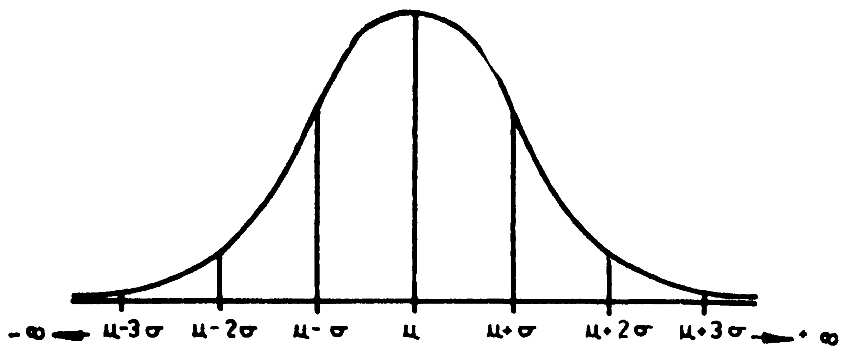


FIG. 1 A TYPICAL NORMAL CURVE

of twice the standard deviation on either side of the mean. For the distance of thrice the standard deviation, the corresponding percentage is 99·73; and

f) If the original observations follow a normal model with mean μ and standard deviation σ, then the averages of random samples of size n drawn from this population also follow a normal model. The mean of the new model (of averages) is same as that of the original model, namely, μ and the standard deviation gets reduced to $\sigma/\sqrt{n}$.

NOTE — These properties have extensive applications in the control chart techniques and statistical tests of significance

5.4 The density function for the normal model is given by:

$$y = \frac{1}{\sqrt{2\pi}\,\sigma} \exp \left\{ \frac{-(x-\mu)^2}{2\,\sigma^2} \right\}$$

$$-\infty < x < +\infty$$

where

y is the ordinate of the curve corresponding to the value x of the variable,

μ is the mean, and

σ is the standard deviation.

5.5 The deviation of the observed value x from the mean measured in the unit of standard deviation, that is, $z = (x - \mu)/\sigma$ is called 'standard normal variate'. In terms of the standard normal variate, the equation of the normal model becomes:

$$y = \frac{1}{\sqrt{2\pi}} \exp \frac{(-z^2)}{2}$$

This model has the mean $= 0$ and the standard deviation $= 1$:

5.5.1 Considering the area under the standard normal curve to be equal to unity, the proportion of area to the left of any given value of the variable has been given in Annex A. This Annex may be used for finding the proportion (or percentage) of the area lying between any two values of the variable.

5.5.2 If it is desired to calculate the proportion of observations that will be less than a specified value x_1, then the standardized variate $z_1 = (x_1 - \mu)/\sigma$ shall be calculated and required proportion to the left of z_1 shall be directly read from Annex A.

5.5.3 If it is desired to calculate the proportion of observations that will be more than a specified value x_2, then the standardized variate $z_2 = (x_2 - \mu)/\sigma$ shall be calculated. The corresponding proportion to the left of z_2 as obtained from Annex A shall be substracted from 1 for getting the required proportion.

5.5.4 If the proportion of observations lying between any two values x_1 and x_2 ($x_2 > x_1$) is required, the respective standardized variates $z_1 = (x_1 - \mu)/\sigma$ and $z_2 = (x_2 - \mu)/\sigma$ shall be computed. The proportions of observations less than z_1 and z_2 shall be read from Annex A and the difference between these two proportions will give the required proportion.

5.5.5 *Example*

The specification limit for weight per unit area of Indian Hessian is given as 299-329 g/m². The mean and standard deviation of the 225 observations on weight per unit area of Indian Hessian are 304·8 g and 7·0 g respectively. Find the percentage of material meeting the specification limits.

The standardized variates are given by:

$$z_1 = \frac{299 - 304{\cdot}8}{7} = \frac{-5{\cdot}8}{7} = -0{\cdot}83$$

and

$$z_2 = \frac{329 - 304{\cdot}8}{7} = \frac{24{\cdot}2}{7} = 3{\cdot}46$$

From Annex A, the area under the normal curve to the left of standardized variate z_1 is 0·203 3. The area under the normal curve to the left of standardized variate z_2 is 0·999 73.

Hence the area under the normal curve between these two standardized variates z_1 and z_2, that is, the proportion of material meeting the specification limits is 0·796 4 or 79·64 percent.

5.5.6 *Example*

The specification limits for tensile strength for LPG cylinders is given as 36-46 kgf/mm². The mean and standard deviation of 200 observations on tensile strength were calculated as 40·5 kgf/mm² and 2·77 kgf/mm² respectively. Find the percentage of LPG cylinders meeting the specified requirements.

The standardized variates are:

$$z_1 = \frac{36 - 40 \cdot 5}{2 \cdot 77} = -1 \cdot 62$$

$$z_2 = \frac{46 - 40 \cdot 5}{2 \cdot 77} = +1 \cdot 99$$

From Annex A, the area under the normal curve to the left of standardized variate z_1 is 0·052 6. The area under the normal curve to the left of standardized variate z_2 is 0·976 7. Hence the area under the normal curve between these two standardized variates z_1 and z_2, that is, the proportion of cylinders meeting the specification limits is 0·924 1 or 92·41 percent.

5.6 Fitting of Normal Model

A manufacturing process produces certain machine bolts. A random sample of 1 000 bolts is selected from a day's production. The diameter of these bolts at the threaded end is measured to the nearest one hundredth of a millimetre and grouped in a frequency distribution as shown in Table 1. Fit a normal model to the above data and test its goodness of fit.

5.6.1 First the sample mean $\bar{x}$ and standard deviation s are calculated from the frequency table. These values are as follows:

$\bar{x} = 10 \cdot 066\ 6$ and $s = 0 \cdot 092$

5.6.2 The next step is to calculate the expected frequencies (E_i) based on the assumption that the above frequency distribution is coming from a normal model. This step is accomplished in Table 2 with classes $-\infty$ and $+\infty$ respectively added at each end Z_1 and Z_2 shown in Table 2 are the standard normal variates for the lower and upper bounds of each class interval. The probability for each class interval is obtained from the table of areas under normal curve (*see* Annex A).

Table 1 Diameter of Bolts (mm)

(*Clause* 5.6)

Class Interval	Frequency
9·745 - 9·795	2
9·795 - 9·845	5
9·845 - 9·895	27
9·895 - 9·945	52
9·945 - 9·995	117
9·995-10·045	203
10·045-10·095	228
10·095-10·145	180
10·145-10·195	105
10·195-10·245	60
10·245-10·295	14
10·295-10·345	4
10·345-10·395	2
10·395-10·445	1
Total	1 000

5.6.3 *Goodness of Fit from* χ^2 *Test*

After calculating the expected frequencies (E_i) for each class interval, their closeness with the observed frequencies (O_i) is tested with the help of χ^2 test by using the following formula:

$$\chi^2 = \sum_i \frac{(O_i - E_i)^2}{E_i}$$

where each of the expected frequency is at least 5. In case some expected frequencies are less

Table 2 Area Under Normal Curve for Each Class Interval

(*Clauses* 5.6.2 *and* 8.5.2)

Class Interval	Z_1	Z_2	Probability ($Z_1 < Z < Z_2$)	E_i (1 000 × 4)
(1)	(2)	(3)	(4)	(5)
$-\infty$ - 9·745	$-\infty$	− 3·49	0	0
9·745 - 9·795	− 3·49	− 2·95	0·001 6	1·6
9·795 - 9·845	− 2·95	− 2·40	0·006 6	6·6
9·845 - 9·895	− 2·40	− 1·86	0·023 2	23 2
9·895 - 9·945	− 1·86	− 1·32	0·062 0	62·0
9·945 - 9·995	− 1·32	− 0·77	0·127 2	127·2
9·995-10·045	− 0·77	− 0·23	0·188 4	188·4
10·045-10·095	− 0·23	0·32	0·216 5	216·5
10·095-10·145	0·32	0·86	0·179 6	179·6
10·145-10·195	0·86	1·40	0·114 1	114·1
10·195-10·245	1·40	1·95	0·055 2	55·2
10·245-10·295	1·95	2·49	0·019 2	19·2
10·295-10·345	2·49	3·03	0·005 2	5·2
10·345-10·395	3·03	3·58	0 001 2	1·2
10·395-10·445	3·58	4·12	0	[illegible]
10·445- $+\infty$	4·12	$+\infty$	[illegible]	
		Tot[illegible]		

than 5, the adjacent classes are pooled so as to make the expected frequency for each class at least 5.

5.6.4 From Table 3, there are 11 classes left after pooling from which the value of χ^2 is calculated. But the degrees of freedom will be only 8 because 2 degrees of freedom are lost for estimating population parameters μ and σ from the sample data and the third degree of freedom for the condition that the sum of expected frequencies must be equal to sum of the observed frequencies. The value of χ^2 for 8 degrees of freedom and at 5 percent level of significance from Annex B is 15·507. Since the calculated value is less than the table value, the null hypothesis is accepted thereby meaning that the sample data has come from a normal model.

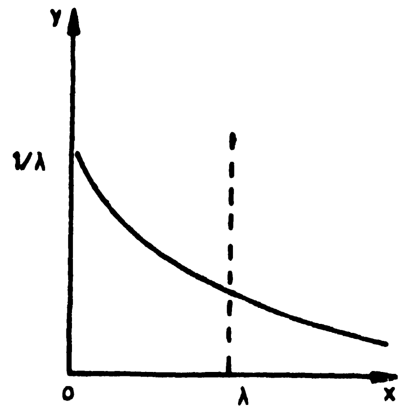


FIG. 2 TYPICAL p. d. f. FOR EXPONENTIAL MODEL

Table 3 Observed and Expected Frequencies

(*Clauses* 5.6.4 *and* 8.5.2)

Class Interval	Observed Frequencies (O_i)	Expected Frequencies (E_i)	($O_i - E_i$)	$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
(1)	(2)	(3)	(4)	(5)	(6)
9·745- 9·795 } 9·795- 9·845 }	7	8·2	−1·2	1·44	0·175 6
9·845- 9·895	27	23·2	3·8	14·44	0·622 4
9·895- 9·945	52	62·0	−10·0	100·00	1·612 9
9·945- 9·995	117	127·2	−10·2	104·04	0·817 9
9·995-10·045	203	188·4	14·6	213·16	1·131 4
10·045-10·095	228	216·5	11·5	132·25	0·610 9
10·095-10·145	180	179·6	0·4	0·16	0·000 9
10·145-10·195	105	114·1	−9·1	82·81	0·725 8
10·195-10·245	60	55·2	4·8	23·04	0·417 4
10·245-10·295	14	19·2	−5·2	27·04	1·408 3
10·295-10·345 } 10·345-10·395 } 10·395-10·445 }	7	6·4	0·6	0 36	0·056 2
				Total	7·579 7

6 EXPONENTIAL MODEL

6.1 This model has extensive applications in life testing and reliability calculations. For this model, the failure rate is constant and is the reciprocal of mean life.

6.2 The probability density function (p. d. f.) for this model is defined as:

$$y = \frac{1}{\lambda} \exp \{ -(x - \gamma)/\lambda \}$$

$$\gamma < x < \infty \quad \text{and} \quad \lambda > 0$$

where γ and ($1/\lambda$) are location and scale parameters respectively. ($1/\lambda$) is also referred as failure rate.

Taking $\gamma = 0$, the p. d. f. of the exponential model is usually defined as:

$$y = (1/\lambda) \exp (- x/\lambda)$$

$$x > 0 \quad \text{and} \quad \lambda > 0$$

6.2.1 A typical form of the p. d. f. of the exponential model is given in Fig. 2.

6.3 Mean and Variance

Mean $= \gamma + \lambda = \lambda$ if γ is taken as 0

Variance $= \lambda^2$

Standard deviation $= \lambda$

Thus for exponential model, the failure rate is the reciprocal of mean life and it is fully specified by its mean. This model is very useful in describing the failure times of complex equipment.

6.4 Tables, for exponential model, have been given in Annex C. Fractional parts of the total area (under the exponential curve) greater than (x/λ) have been tabulated. Thus, for example, if (x/λ) = 0·45, the probability of a value greater than (x/λ) is 0·637 6. It may also be noted that for the exponentially distributed population 36·8 percent of the values will be above the average and 63·2 percent below the average.

6.4.1 *Example*

Results of sample measurement indicate that for a particular equipment the mean time between failures (commonly known as MTBF) is found to be 100 hours. What is the probability that the time between two successive failures of this equipment will be at least 5 hours.

The problem is to find the area under the curve beyond 5 hours

Here λ = 100 hours (x/λ) = 5/100 = 0·05

Corresponding to (x/λ) = 0·05, the area from 0·05 to ∞ from Annex C is 0·951 2, that is, 95·12 percent. Therefore, the chance that the equipment will operate without failure continuously for 5 hours or more is 95·12 percent.

6.5 Fitting an Exponential Model

Before fitting an exponential model to a given data, it is necessary as a first step to examine whether mean and standard deviation calculated from the data are approximately of the same order. There is no point in fitting an exponential model if the mean and standard deviation differ widely. Once a model has been fitted, it is essential to carry out an exact test for goodness of fit.

6.5.1 *Example*

The following table gives the distribution of demand for samples for a two-month period. Fit an exponential model to the data and also test its goodness of fit.

Number of Units Demanded (x)	*Observed Frequency* (O)
0	8
1	8
2	5
3	4
4	4
5	3
6	2
7	2
Total	36

From the above data,

$\bar{x}$ = 2·42 and s = 2·13

As $\bar{x}$ and s are approximately of same order, one can go for the actual fitting of data.

As mean = λ = 2·42, ($1/\lambda$) = 0·41 and the density function is:

$$y = 0{\cdot}41 \exp (- 0{\cdot}41x),\ x > 0$$

The probabilities for different values of x are calculated. Multiplying these probabilities by the total frequency, that is, 36, the expected frequencies are obtained (*see* Table 4).

The closeness of expected frequencies with the observed frequencies is tested by using χ^2— test (*see* Table 5).

Table 4 Observed and Expected Frequencies

(*Clause* 6.5.1)

No. of Units Demanded, X	Observed Frequency, O	Expected Frequency, E
(1)	(2)	(3)
0	8	12
1	8	8
2	5	5
3	4	4
4	4	2
5	3	2
6	2	1
7	2	2

Table 5 Calculations for the χ^2 - - Test

(*Clause* 6.5.1)

X	O		E		$(O-E)$	$(O-E)^2$	$(O-E)^2/E$
(1)	(2)		(3)		(4)	(5)	(6)
0	8		12		−4	16	1·33
1	8		8		0	0	0
2	5		5		0	0	0
3	4	8	4	6	2	4	0·67
4	4		2				
5	3	7	2	5	2	4	0·80
6	2		1				
7	2		2				
						χ^2 =	2·80

From Table 5, the total number of classes after pooling are 5. Two degrees of freedom are apportioned for the total frequency and the estimation of the mean. Thus, χ^2 = 2.80 is compared with the tabulated value of χ^2 for 3 degrees of freedom which is 7·82 at 5 percent level of significance (*see* Annex B). Since calculated value of χ^2 is less than the tabulated value, the fit can be taken as a good one.

6.6 Reliability Estimation

The reliability of a unit (or a system) is defined as the probability that it will perform satisfactorily atleast for a specified period of time, when used in the manner and for the purpose intended, without a major breakdown. If X is the life time of the unit, the reliability of the unit at time t is given by:

$$R(t) = \text{Prob} (X > t) = 1 - F(t)$$

where $F(t)$ is the distribution function of the failure time and is defined as:

$$F(t) = \text{Prob} (X < t) = \int_0^t f(x)\, dx$$

where $f(x)$ is the p.d.f. of a given model.

For exponential distribution:

$$R(t) = \exp (- t/\bar{x})$$

where $\bar{x}$ = mean life

6.6.1 *Example*

A manu[illegible]

of elect[illegible]

purpose, a random sample of 20 tubes is put to test and their failure times (in hours) are given below:

9·9	35·6	57·9	94·6	141·4
154·4	163·3	226·7	244·3	337·2
391·8	417·2	444·6	461·2	497·1
582·6	606·8	616·3	672·0	784·7

Assuming that the failure rate is constant (exponential model), he wants to find the probability that an electronic tube will survive for at least 1 000 h. For this purpose, the average is first calculated and then reliability.

$$\bar{x} = \sum_{i=1}^{20} x_i/n = 346{\cdot}98$$

$$R(t) = \exp(-t/\bar{x})$$

At t = 1 000 hours

$$R(t) = \exp(-1\,000/346{\cdot}98) = 0{\cdot}056$$

Therefore, the probability that an electronic tube will survive for at least 1 000 h is only 5·6 percent.

6.7 Reliability Estimation with Censored Samples

6.7.1 *General*

In many practical situations it will not be possible to carry out life testing experiments on all the samples as these are usually destructive. In such cases, the experiment may be terminated either when a pre-assigned number of items, say r ($< n$) have failed (known as failure-censored samples) or the experiment may be terminated after a pre-assigned time (known as time-censored samples).

6.7.2 *Failure — Censored Samples*

Let n items were put to life test experiment and it was terminated when $r (< n)$ items failed. Let the failure times of r items be $x_1 < x_2 < \ldots\ldots < x_r$ and $(n - r)$ items survived until time x_r. The items that failed may or may not be replaced:

a) *Without replacement*

The maximum likelihood estimate of λ, a parameter of exponential model when failure items are not replaced, is given by:

$$\hat{\lambda} = [\sum_{i=1}^{r} x_i + (n - r)\, x_r]/r$$

b) *With replacement*

The maximum likelihood estimate of λ is given by:

$$\hat{\lambda} = (n\, x_r)/r$$

Reliability function, $R(t) = \exp(-t/\hat{\lambda})$

6.7.3 *Example*

60 items were placed on test and the test was terminated after the first 10 items failed. The failure time (in hours) were recorded as follows:

85	151	280	376	492
520	623	715	820	914

Assuming the failure time distribution to be exponential, estimate the parameter of exponential model and also the reliability at t = 600 hours, if the failed items are:

a) not replaced, and

b) replaced.

In this example,

n = 60, and r = 10

When the items are not replaced,

$$\hat{\lambda} = (\sum_{i=1}^{10} x_i + 50\, x_{10})/10 = 5\,068 \text{ hours}$$

$$R(600) = \exp(-600/5\,068) = 0{\cdot}888\,7$$

When the items are replaced,

$$\hat{\lambda} = n\, x_{10} = 5\,484 \text{ hours}$$

$$R(600) = \exp(-600/5\,484) = 0{\cdot}896\,7$$

6.7.4 *Time Censored Samples*

Let there be m items that failed before stipulated time (t_0) and the failure times of these m items be $x_1 < x_2 \ldots\ldots < x_m$. Let the items that failed are not replaced. The maximum likelihood estimate of λ is given by:

$$\hat{\lambda} = \{\sum_{i=1}^{m} x_i + (n - m)\, t_0\}/m \qquad m > 0$$

$$= n\, t_0, \; m = 0, \text{ and}$$

$$R(t) = \exp(-t/\hat{\lambda})$$

7 GAMMA MODEL

7.1 In accordance with the parallel strand-theory where each component consists of many subcomponents in the manner of a multi-strand rope, the characteristic life pattern of the component is the sum of the characteristic life patterns of all sub-components. If the subcomponents follow the exponential model, then the main component can be expected to follow the gamma model.

7.2 The gamma model is actually the model of sum of n, identical and independent exponential variables. Thus the probability density function for the gamma model can be written as:

$$y = \frac{(x/\lambda)^{n-1}\, e^{-x/\lambda}}{\lambda(n-1)!} \qquad 0 < x < \infty, \quad \lambda > 0$$

When n = 1, it reduces to exponential model discussed in 6.

Typical forms of the probability density function of the gamma model are given in Fig. 3 (for $n = 1/2$, 1 and 3).

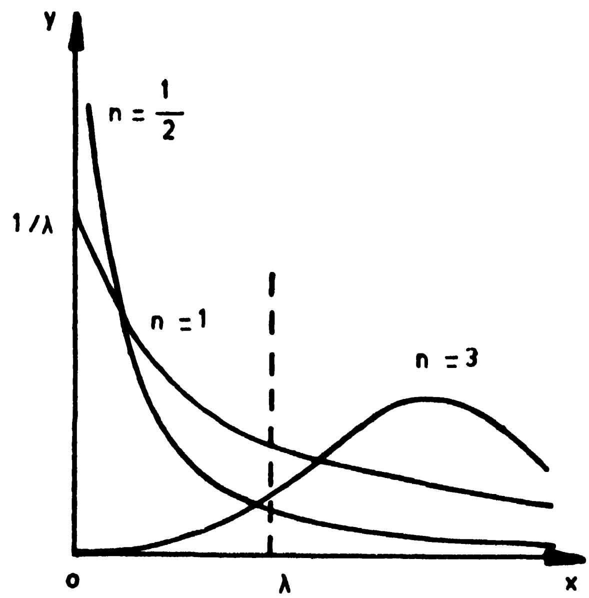


FIG. 3 TYPICAL p.d.f. OF THE GAMMA MODEL

NOTE — Although gamma model has been obtained as the sum of n identical and independent exponential variables, it can be generalized to non-integral values of n also. Its generalized form can then be written as:

$$y = \frac{(x/\lambda)^{n-1} \exp\{-(x/\lambda)\}}{\lambda \Gamma n}$$

where

$\Gamma n = (n - 1)\,\Gamma(n - 1)$

$= (n - 1)!$ for all integral values.

$\Gamma 1 \quad 0! = 1$; $\Gamma(1/2)$ − and $\Gamma 0 = 1$

7.3 Mean and Variance

$E(X) = n\lambda.$

$V(X) = n\lambda^2$, and

Standard deviation $= \sqrt{n}\,\lambda.$

7.4 For the convenience of preparing tables and charts for the gamma model, it has been standardized. The standardized random variable is defined as:

$$u = x/\lambda$$

Thus the probability density function is given by:

$$\frac{e^{-u} u^{n-1}}{\Gamma n} \qquad 0 < u < \infty,$$

$E(u) = n$ and $V(u) = n$, and

Standard deviation $= \sqrt{n}$.

For $n = 1$, the model is called standardized exponential model.

8. LOG NORMAL MODEL

8.1 This model has been used to approximate wear out failures when the failure rate increases with time. Suppose the characteristic life pattern of the component is taken as the size of the fatigue crack at the successive stages of its growth. Assuming the proportional effect theory of failures wherein the crack growth at any stage is proportional to the crack size at the preceding stage for all stages, the size of the crack can be expected to follow the lognormal model.

8.2 If any variable X is lognormally distributed, then $\log_n X$ is distributed normally.

The density function for lognormal model is given by:

$$y = \frac{1}{x\sigma\sqrt{2\pi}} \exp\{-(\log_n X - \mu)^2/2\sigma^2\}$$

where $\sigma > 0$ and $0 < X < \infty = 0$ elsewhere

where μ and σ are the mean and standard deviation of the transformed characteristic $\log_n X$.

8.2.1 Typical forms of the probability density function of lognormal model are given in Fig. 4 (for $\sigma = 0{\cdot}3$, $1{\cdot}0$ and $1{\cdot}5$)

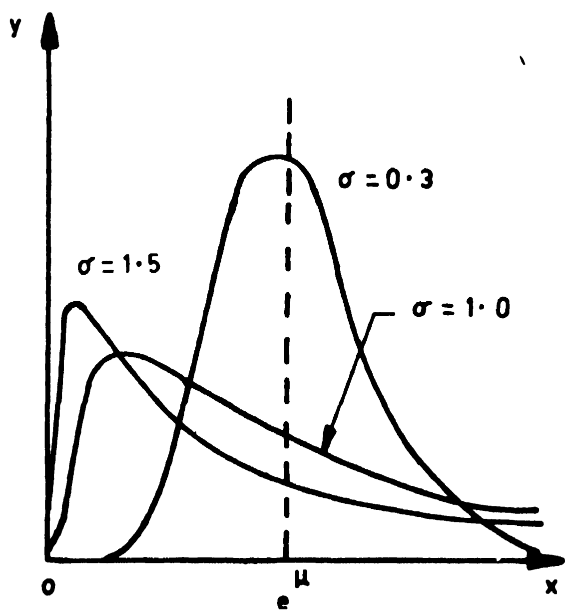


FIG. 4 TYPICAL p.d.f. OF THE LOGNORMAL MODEL

8.3 Mean and Variance

$$\text{Mean} = \exp\left(\mu + \frac{\sigma^2}{2}\right)$$

Variance $= [\exp(2\mu + \sigma^2)]\,[\exp(\sigma^2 - 1)]$

Standard deviation

$$= \sqrt{[\exp \frac{(2\mu + \sigma^2)}{2}]\,[\exp(\sigma^2 - 1)]}$$

8.4 The fitting of a lognormal to the data can be done by first finding the natural logarithms of the given variable X and then fitting a normal model to these values. A χ^2 test can be carried out to examine the goodness of fit. Further, the mean and variance of the variable X can be calculated from the mean μ and standard deviation σ of $\log_n X$ by the formula given in **8.3**.

8.5 Example

A new control device was tested on 50 diesel locomotives. Whenever a device failed, the distance was recorded and the device was returned to the factory for failure analysis. The distance of each device is given in Table 6. The underlined failure mechanism suggested a lognormal model for time to failure. Test whether the data follows lognormal model and also obtain its parameters.

Table 6 Distance in Million Metres to Failure

(*Clause* 8.5)

36·6	40·4	44·7	49·4	54·6
60·3	63·4	66·7	73·7	77·5
81·5	83·1	85·6	90·0	92·8
94·6	99·5	104·6	108·8	111·1
113·3	115·6	117·9	120·3	125·2
127·7	130·3	133·0	135·6	138·4
141·2	144·0	164·0	169·0	174·2
179·5	184·9	190·6	196·4	198·3
212·7	225·9	239·8	254·7	267·7
284·3	314·2	347·2	383·8	403·4

8.5.1 As mentioned in **8.4**, the natural logarithms of these distances shall be first obtained and then a frequency table prepared (*see* Table 7).

Table 7 Frequency Table for Natural Logarithms of Distances

(*Clause* 8.5.1)

Class Intervals	Frequency
3·5-3·8	2
3·8-4·1	3
4·1-4·4	5
4·4-4·7	9
4·7-5·0	13
5·0-5·3	8
5·3-5·6	5
5·6-5·9	3
5·9-6·2	2
Total	50

8.5.2 The next step is to calculate the expected frequencies as calculated in **5.6.2** (*see also* Tables 2 and 3). The expected frequencies so calculated are given in Table 8.

Table 8 Expected Frequencies

(*Clause* 8.5.2)

Class Interval	Z_1	Z_2	Probability $(Z_1 < Z < Z_2)$	E_i
3·5-3·8	$-\infty$	$-1·79$	0·036 7	1·8
3·8-4·1	$-1·79$	$-1·28$	0·063 6	3·2
4·1-4·4	$-1·28$	$-0·76$	0·123 3	6·2
4·4-4·7	$-0·76$	$-0·24$	0·181 6	9·1
4·7-5·0	$-0·24$	$+0·28$	0·205 1	10·2
5·0-5·3	$+0·28$	$+0·79$	0·174 9	8·7
5·3-5·6	$+0·79$	$+1·31$	0·119 7	6·0
5·6-5·9	$+1·31$	$+1·83$	0·061 5	3·1
5·9-6·2	$+1·83$	$+2·34$	0·024 0	1·1
6·2 and above	$+2·34$	$+\infty$	0·010 0	0·5

8.5.3 *Goodness of Fit*

The value of χ^2 calculated for the example is 1·12 (*see* Table 9). The table value of χ^2 for 3 degrees of freedom at 5 percent level of significance is 7·82 (*see* Annex B). Since the calculated value is less than the table value, the null hypothesis that the data has come from a lognormal model is accepted.

9 WEIBULL MODEL

9.1 This model has extensive applications in reliability testing of complex items. The failure time of complex items, when plotted against time, generally assumes the shape of 'bath-tub' curve with 3 distinct phases, namely, debugging phase, chance failure phase and wear out phase. The Weibull model is capable of describing all these phases by taking appropriate values of the different parameters.

9.2 The Weibull is a family of models having the general density function as:

$$y = \beta \lambda^{\beta} (x - \gamma)^{\beta-1} \exp \{ - \lambda (x - \gamma)\}^{\beta}$$

where

λ is the scale parameter ($\lambda > 0$),

β is the shape parameter ($\beta > 0$), and

γ is the location parameter ($\gamma < x < \infty$).

Table 9 Observed and Expected Frequencies

(*Clause* 8.5.3)

Observed Frequencies (O_i)		Expected Frequencies (E_i)		$(O_i - E_i)$	$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
2	5	1·8	5·0	0	0	0
3		3·2				
5		6·2		−1·2	1·44	0·23
9		9·1		−0·1	0·01	0
13		10·2		2·8	7·84	0·77
8		8·7		−0·7	0·49	0·06
5	10	6·0	10·8	−0·8	0·64	0·06
3		3·1				
2		1·2				
0		0·5				
					Total =	1·12

9.3 The curve varies greatly depending on the values of these parameters. The location parameters is the smallest possible value of X. This is usually taken as 0 thereby simplifying the density function as:

$$y = \beta \lambda^{\beta} x^{\beta-1} \exp \{ -(\lambda x)^{\beta} \}, 0 < x < \infty$$

The shape parameter β reflects the pattern of the curve. When $\beta = 1$, the Weibull model reduces to exponential model. When β is about 3·5 and $\lambda = 1$, the Weibull model approximates to normal model. The ability of this model has made it increasingly popular in practice because it reduces the problem of examining a set of data and deciding which of the several common models, like normal or exponential, would be most appropriate.

9.3.1 Typical forms of the probability density function of the Weibull model are given in Fig. 5 (for $\beta = 1/2$, 1 and 4).

9.4 Mean and Variance

$$\text{Mean} = \gamma + \left(\frac{1}{\lambda}\right) \Gamma\left(\frac{1+\beta}{\beta}\right)$$

$$= \frac{1}{\lambda} \Gamma\left(\frac{1+\beta}{\beta}\right)$$, When γ is taken as 0, and

Variance =

$$\frac{1}{\lambda^2}\left[\Gamma\left(\frac{2+\beta}{\beta}\right) - \left\{\Gamma\left(\frac{1+\beta}{\beta}\right)\right\}^2\right]$$

Standard deviation

$$= \frac{1}{\lambda}\left[\Gamma\left(\frac{2+\beta}{\beta}\right) - \left\{\Gamma\left(\frac{1+\beta}{\beta}\right)\right\}^2\right]^{1/2}$$

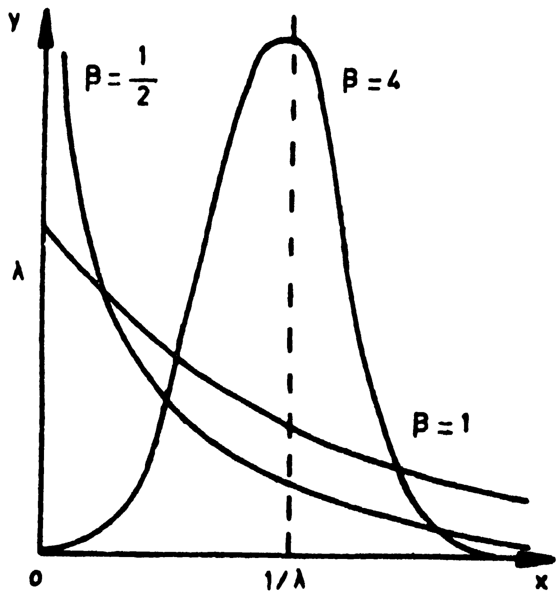


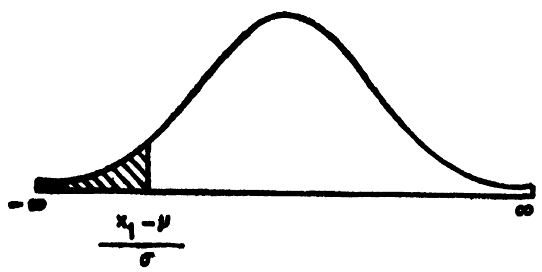
FIG. 5 TYPICAL p.d.f. OF THE WEIBULL MODEL

ANNEX A

(*Clauses* 5.5.1, 5.5.2, 5.5.3, 5.5.4, 5.5.5, 5.5.6 *and* 5.6.2)

AREAS UNDER THE NORMAL CURVE

Proportion of total area under the curve left of $\frac{x_1 - \mu}{\sigma}$ (x_1 represents any desired value of the variable x)



NORMAL CURVE

The shaded portion is the area which is given in the table.

$\frac{x_1 - \mu}{\sigma}$	0·09	0·08	0·07	0·06	0·05	0·04	0·03	0·02	0·01	0·00
−3·5	0·000 17	0·000 17	0·000 18	0·000 19	0·000 19	0·000 20	0·000 21	0·000 22	0·000 22	0·000 23
−3·4	0·000 24	0·000 25	0·000 26	0·000 27	0·000 28	0·000 29	0·000 30	0·000 31	0 000 33	0·000 34
−3·3	0·000 35	0·000 36	0·000 38	0·000 39	0·000 40	0·000 42	0·000 43	0·000 45	0·000 47	0·000 48
−3·2	0·000 50	0·000 52	0·000 54	0·000 56	0·000 58	0·000 60	0·000 62	0·000 64	0·000 66	0·000 69
−3·1	0·000 71	0·000 74	0·000 76	0·000 79	0·000 82	0·000 85	0·000 87	0·000 90	0·000 94	0·000 97
−3·0	0·001 00	0·001 04	0·001 07	0·001 11	0·001 14	0·001 18	0·001 22	0·001 26	0·001 31	0·001 35
−2·9	0·001 4	0·001 4	0·001 5	0·001 5	0·001 6	0·001 6	0·001 7	0·001 7	0·001 8	0·001 9
−2·8	0·001 9	0·002 0	0·002 1	0·002 1	0·002 2	0·002 3	0 002 3	0·002 4	0·002 5	0·002 6
−2·7	0·002 6	0·002 7	0·002 8	0·002 9	0·003 0	0·003 1	0·003 2	0·003 3	0·003 4	0·003 5
−2·6	0·003 6	0·003 7	0·003 8	0·003 9	0·004 0	0·004 1	0·004 3	0·004 4	0·004 5	0·0 4 7
−2·5	0·004 8	0·004 9	0·005 1	0·005 2	0·005 4	0·005 5	0·005 7	0·005 9	0·006 0	0·006 2
−2·4	0·006 4	0·006 6	0·006 8	0·006 9	0·007 1	0·007 3	0·007 5	0·007 8	0·008 0	0·008 2
−2·3	0·008 4	0·008 7	0·008 9	0·009 1	0·009 4	0·009 6	0·009 9	0·010 2	0·010 4	0·010 7
−2·2	0·011 0	0·011 3	0·011 6	0·011 9	0·012 2	0·012 5	0·012 9	0·013 2	0·013 6	0·013 9
−2·1	0·014 3	0·014 6	0·015 0	0·015 4	0·015 8	0·016 2	0·016 6	0·017 0	0·017 4	0·017 9
−2·0	0·018 3	0·018 8	0·019 2	0·019 7	0·020 2	0·207 2	0·021 2	0·021 7	0·022 2	0·022 8
−1·9	0·023 3	0·023 9	0·024 4	0·025 0	0·025 6	0·026 2	0·026 8	0·027 4	0·028 1	0·028 7
−1·8	0·029 4	0·030 1	0·030 7	0·031 4	0·032 2	0·032 9	0·033 6	0·034 4	0·035 1	0·035 9
−1·7	0·036 7	0·037 5	0·038 4	0·039 2	0·040 1	0·040 9	0·041 8	0·042 7	0·043 6	0·044 6
−1·6	0·045 5	0·046 5	0·047 5	0·048 5	0·049 5	0·050 5	0·051 6	0·052 6	0·053 7	0·054 8
−1·5	0·055 9	0·057 1	0·058 2	0·059 4	0·060 6	0·061 8	0·063 0	0·064 3	0·065 2	0·066 8
−1·4	0 068 1	0·069 4	0·070 8	0·072 1	0·073 5	0 074 9	0·076 4	0·077 8	0·079 3	0·080 8
−1·3	0·082 3	0·083 8	0·085 3	0·086 9	0·088 5	0·090 1	0·091 8	0·093 4	0·095 1	0·096 8
−1·2	0·098 5	0·100 3	0·102 0	0·103 8	0·105 7	0·107 5	0·109 3	0·111 2	0·113 1	0·115 1
−1·1	0·117 0	0·119 0	0·121 0	0·123 0	0·125 1	0·127 1	0·129 2	0·131 4	0·133 5	0·135 7
−1·0	0·137 9	0·140 1	0·142 3	0·144 6	0·146 9	0·149 2	0·151 5	0·153 9	0·156 2	0·158 7
−0·9	0·161 1	0·163 5	0·166 0	0·168 5	0·171 1	0·173 6	0·176 2	0·178 8	0·181 4	0·184 1
−0·8	0·186 7	0·189 4	0·192 2	0·194 9	0·197 7	0·200 5	0·203 3	0·206 1	0·209 0	0·211 9
−0·7	0·214 8	0·217 7	0·220 7	0·223 6	0·226 6	0·229 7	0·232 7	0·235 8	0·238 9	0·242 0
−0·6	0·245 1	0·248 3	0·251 4	0·254 6	0·257 8	0·261 1	0·264 3	0·267 6	0·270 9	0·274 3
−0·5	0·277 6	0·281 0	0·284 3	0·287 7	0·291 2	0·294 6	0·298 1	0·305 1	0·305 0	0·308 5
−0·4	0·312 1	0·315 6	0·319 2	0·322 8	0·326 4	0·330 0	0·333 6	0·337 2	0·340 9	0·344 6
−0·3	0·348 3	0·352 0	0·355 7	0·359 4	0·363 2	0·366 9	0·370 7	0·374 5	0·378 3	0·382 1
−0·2	0·385 9	0·389 7	0·393 6	0·397 4	0·401 3	0·405 2	0·409 0	0·412 9	0·416 8	0·420 7
−0·1	0·424 7	0·428 6	0·432 5	0·436 4	0·440 4	0·444 3	0·448 3	0·452 2	0·456 2	0·460 2
−0·0	0·464 1	0·468 1	0·472 1	0·476 1	0·480 1	0·484 0	0·488 0	0·492 0	0·496 0	0·500 0

(*Continued*)

AREAS UNDER THE NORMAL CURVE — *Contd*

$\frac{x_1 - \mu}{\sigma}$	0·00	0·01	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09
+0·0	0·500 0	0·504 0	0·508 0	0·512 0	0·516 0	0·519 9	0·523 9	0·527 9	0·531 9	0·535 9
+0·1	0·539 8	0·543 8	0·547 8	0·551 7	0·555 7	0·559 6	0·563 6	0·567 5	0·571 4	0·575 3
+0·2	0·579 3	0·583 2	0·587 1	0·591 0	0·594 8	0·598 7	0·602 6	0·606 4	0·610 3	0·614 1
+0·3	0·617 9	0·621 7	0·625 5	0·629 3	0·633 1	0·636 8	0·640 6	0·644 3	0 648 0	0·651 7
+0·4	0·655 4	0·659 1	0·662 8	0·666 4	0·670 0	0·673 6	0·677 2	0·680 8	0·684 4	0·687 9
+0·5	0·691 5	0·695 0	0·698 5	0·701 9	0·705 4	0·708 8	0·712 3	0·715 7	0·719 9	0·722 4
+0·6	0·725 7	0·729 1	0·732 4	0·735 7	0·738 9	0·742 2	0·745 4	0·748 6	0·751 7	0·754 9
+0·7	0·758 0	0·761 1	0·764 2	0·767 3	0·770 4	0·773 4	0·776 4	0·779 4	0·782 3	0·785 2
+0·8	0·788 1	0·791 0	0·793 9	0·796 7	0·799 5	0·802 3	0·805 1	0·807 9	0·810 6	0·813 3
+0·9	0·815 9	0·818 6	0·821 2	0·823 8	0·826 4	0·828 9	0·831 5	0.834 0	0·836 5	0·838 9
+1·0	0·841 3	0·843 8	0·846 1	0·848 5	0·850 8	0·853 1	0·855 4	0·857 7	0·859 9	0·862 1
+1·1	0·864 3	0·866 5	0·868 6	0 870 8	0·872 9	0·874 9	0·877 0	0·879 0	0·881 0	0·883 0
+1·2	0.884 9	0·886 9	0·888 8	0·890 7	0·892 5	0·894 4	0·896 2	0·898 0	0·899 7	0·901 5
+1·3	0·903 2	0·904 9	0·906 6	0·908 2	0·909 9	0·911 5	0·913 1	0·914 7	0·916 2	0·917 7
+1·4	0·919 2	0·920 7	0·922 2	0·923 6	0·925 1	0·926 5	0·927 9	0·929 2	0·930 6	0·931 9
+1·5	0·933 2	0·934 5	0·935 7	0·937 0	0·938 3	0·939 4	0·940 6	0·941 8	0·942 9	0·944 1
+1·6	0·945 2	0·946 3	0·947 4	0·948 4	0·949 5	0·950 5	0·951 5	0·952 5	0·953 5	0·954 5
+1·7	0·955 4	0·956 4	0·957 3	0·958 2	0·959 1	0·959 9	0·960 8	0·961 6	0·962 5	0·963 3
+1·8	0·964 1	0·964 9	0·965 6	0·966 4	0·967 1	0·967 8	0·968 6	0·969 3	0·969 9	0·970 6
+1·9	0·971 3	0.971 9	0·972 6	0·973 2	0·973 8	0·974 4	0·975 0	0·975 6	0·976 1	0·976 7
+2·0	0·977 3	0·977 8	0·978 3	0·978 8	0·979 3	0·979 8	0·980 3	0·980 8	0·981 2	0·981 7
+2·1	0·982 1	0·982 6	0·983 0	0·983 4	0·983 8	0·984 2	0·984 6	0·985 0	0·985 4	0·985 7
+2·2	0·986 1	0·986 4	0·986 8	0·987 1	0·987 5	0·987 8	0·988 1	9·988 4	0·988 7	0·989 0
+2·3	0·989 3	0·989 6	0·989 8	0·990 1	0·990 4	0·990 6	0·990 9	0·991 1	0·991 3	0·991 6
+2·4	0·991 8	0·992 0	0·992 2	0·992 5	0·992 7	0·992 9	0·993 1	0·993 2	0·993 4	0·993 6
+2·5	0·993 8	0·994 0	0·994 1	0·994 3	0·994 5	0·994 6	0·994 8	0·994 9	0·995 1	0·995 2
+2·6	0·995 3	0·995 5	0·995 6	0·995 7	0·995 9	0·996 0	0·996 1	0·996 2	0·996 3	0·996 4
+2·7	0·996 5	0·996 6	0·996 7	0·996 8	0·996 9	0·997 0	0·997 1	0·997 2	0·997 3	0·997 4
+2·8	0·997 4	0·997 5	0·997 6	0·997 7	0·997 7	0·997 8	0·997 9	0·997 9	0·998 0	0·998 1
+2·9	0·998 1	0·998 2	0·998 3	0·998 3	0·998 4	0·998 4	0 998 5	0·998 5	0·998 6	0·998 6
+3·0	0·998 65	0·998 69	0·998 74	0·998 78	0·998 82	0·998 86	0·998 89	0·998 93	0·998 96	0·999 00
+3·1	0·999 03	0·999 06	0·999 10	0·999 13	0·999 15	0·999 18	0·999 21	0·999 24	0·999 26	0·999 29
+3·2	0·999 31	0·999 34	0·999 36	0·999 38	0 999 40	0·099 42	0·999 44	0·999 46	0·999 48	0·999 50
+3·3	0·999 52	0·999 53	0·999 55	0·999 57	0·999 58	0·999 60	0·999 61	0·999 62	0·999 64	0·999 65
+3·4	0·999 66	0·999 67	0·999 69	0·999 70	0·999 71	0·999 72	0·999 73	0·999 74	0·999 75	0·999 76
+3·5	0·999 77	0·999 78	0·999 78	0·999 79	0·999 80	0·999 81	0·999 81	0·999 82	0·999 83	0·999 83

ANNEX B

(*Clauses* 5.6.4, 6.5.1 *and* 8.5.3)

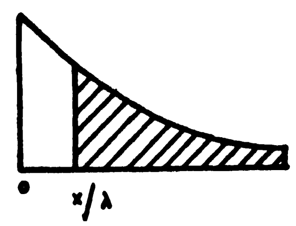
CRITICAL VALUES OF χ^2-DISTRIBUTION

Degrees of Freedom	*Significance Level*		*Degrees of Freedom*	*Significance Level*	
	0·05	0·01		0·05	0·01
(1)	(2)	(3)	(1)	(2)	(3)
1	3·84	6·64	20	31·41	37·57
2	5·99	9·21	21	32·67	38·93
3	7·82	11·34	22	33·92	40·29
4	9·49	13·28	23	35·17	41·64
5	11·07	15·09	24	36·42	42·98
6	12·59	16·81	25	37·65	44·31
7	14·07	18·48	26	38·89	45·64
8	15·51	20·09	27	40·11	46·96
9	16·92	21·67	28	41·34	48·28
10	18·31	23·21	29	42·56	49·59
11	19·68	24·73	30	43·77	50·89
12	21·03	26·22	40	55·75	63·69
13	22·36	27·69	50	67·50	76·15
14	23·69	29·14	60	79·08	88·38
15	25·00	30·58	70	90·53	100·42
16	26·30	32·00	80	101·88	112·33
17	27·59	33·41	90	113·14	124·12
18	28·87	34·81	100	124·34	135·81
19	30·14	36·19			

ANNEX C

(*Clauses* 6.3 *and* 6.3.1)

PROPORTION OF THE AREA UNDER EXPONENTIAL DISTRIBUTION TO THE RIGHT OF THE VALUE X/λ



λ is the mean value

x/λ	0·00	0·01	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09
0·0	1·000 0	0·990 0	0·980 2	0·970 4	0·960 8	0·951 2	0·941 8	0·932 4	0·923 1	0·913 9
0·1	0·904 8	0·895 8	0·886 0	0·878 1	0·861 4	0·869 4	0·852 1	0·843 7	0·835 3	0·827 0
0·2	0·818 7	0·810 6	0·802 5	0·794 5	0·786 6	0·778 8	0·771 1	0·763 4	0·775 8	0·748 3
0·3	0·740 8	0·733 4	0·726 1	0·718 9	0·711 8	0·704 7	0.697 7	0·690 7	0·683 9	0·677 1
0·4	0·670 3	0·663 7	0·657 0	0·650 5	0·644 0	0·637 6	0·631 3	0·625 0	0·618 8	0·612 6
0·5	0·606 5	0·600 5	0·594 5	0·588 6	0·582 7	0·576 9	0·571 2	0·565 5	0·559 9	0·554 3
0·6	0·548 8	0·543 4	0·557 9	0·532 6	0·527 3	0·522 0	0·516 9	0·511 7	0·506 6	0·501 6
0·7	0·496 6	0·491 6	0 486 8	0·481 9	0·477 1	0·472 4	0·467 7	0·463 0	0·458 4	0·453 8
0·8	0·449 3	0·444 9	0·440 4	0·436 0	0·431 7	0·427 4	0·423 4	0·419 0	0·414 8	0·410 7
0·9	0·406 6	0·402 5	0·398 5	0·394 6	0·390 6	0·386 7	0·382 9	0·379 1	0·375 3	0·371 6
	0·0	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9
1·0	0·367 9	0·332 9	0·301 2	0·272 5	0·246 6	0·223 1	0·201 9	0·182 7	0·165 3	0·149 6
2·0	0·135 3	0·122 5	0·110 8	0·100 3	0·090 7	0·082 1	0·074 3	0·067 2	0·060 8	0·055 0
3·0	0·049 8	0·045 0	0·040 8	0·036 9	0·033 4	0·030 2	0·027 3	0·024 7	0·022 4	0·020 2
4·0	0·018 3	0·016 6	0·015 0	0·013 0	0·012 3	0·011 1	0·010 1	0·009 1	0·008 2	0·007 4
5·0	0·006 7	0·006 1	0·005 5	0·005 0	0·004 5	0·004 1	0·003 7	0·003 3	0·003 0	0·002 7
6·0	0·002 5	0·002 2	0·002 0	0·001 8	0·001 7	0·001 5	0·001 4	0·001 2	0·001 1	0·001 0

Bureau of Indian Standards

BIS is a statutory institution established under the *Bureau of Indian Standards Act*, 1986 to promote harmonious development of the activities of stardardization, marking and quality certification of goods and attending to connected matters in the country.

Copyright

Review of Indian Standards

Amendments are issued to standards as the need arises on the basis of comments. Standards are also reviewed periodically; a standard along with amendments is reaffirmed when such review indicates that no changes are needed; if the review indicates that changes are needed, it is taken up for revision. Users of Indian Standards should ascertain that they are in possession of the latest amendments or edition by referring to the latest issue of 'BIS Handbook' and 'Standards: Monthly Additions'.

This Indian Standard has been developed from Doc: No. STAT/EC 3 (268)

Amendments Issued Since Publication

Amend No.	Date of Issue	Text Affected

BUREAU OF INDIAN STANDARDS

Headquarters:

Manak Bhavan, 9 Bahadur Shah Zafar Marg, New Delhi 110002
Telephones: 323 01 31, 323 3375, 323 94 02

Telegrams: Manaksanstha
(Common to all offices)

Regional Offices:		Telephone
Central	: Manak Bhavan, 9 Bahadur Shah Zafar Marg NEW DELHI 110002	323 76 17, 323 38 41
Eastern	: 1/14 C.I.T. Scheme VII M, V.I.P. Road, Kankurgachi CALCUTTA 700054	337 84 99, 337 85 61 337 86 26, 337 91 20
Northern	: SCO 335-336, Sector 34-A, CHANDIGARH 160022	60 38 43 60 20 25
Southern	: C.I.T. Campus, IV Cross Road, CHENNAI 600113	254 12 16, 254 14 42 254 25 19, 254 13 15
Western	: Manakalaya, E9 MIDC, Marol, Andheri (East) MUMBAI 400093	832 92 95, 832 78 58 832 78 91, 832 78 92

Branches : AHMEDABAD. BANGALORE. BHOPAL. BHUBANESHWAR. COIMBATORE. FARIDABAD. GHAZIABAD. GUWAHATI. HYDERABAD. JAIPUR. KANPUR. LUCKNOW. NAGPUR. NALAGARH, PATNA. PUNE. RAJKOT. THIRUVANANTHAPURAM.

Printed at Simco Printing Press, Delhi